Polar Plots at Low Frequencies: The Acoustic Centre

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co-conspirator David Henwood

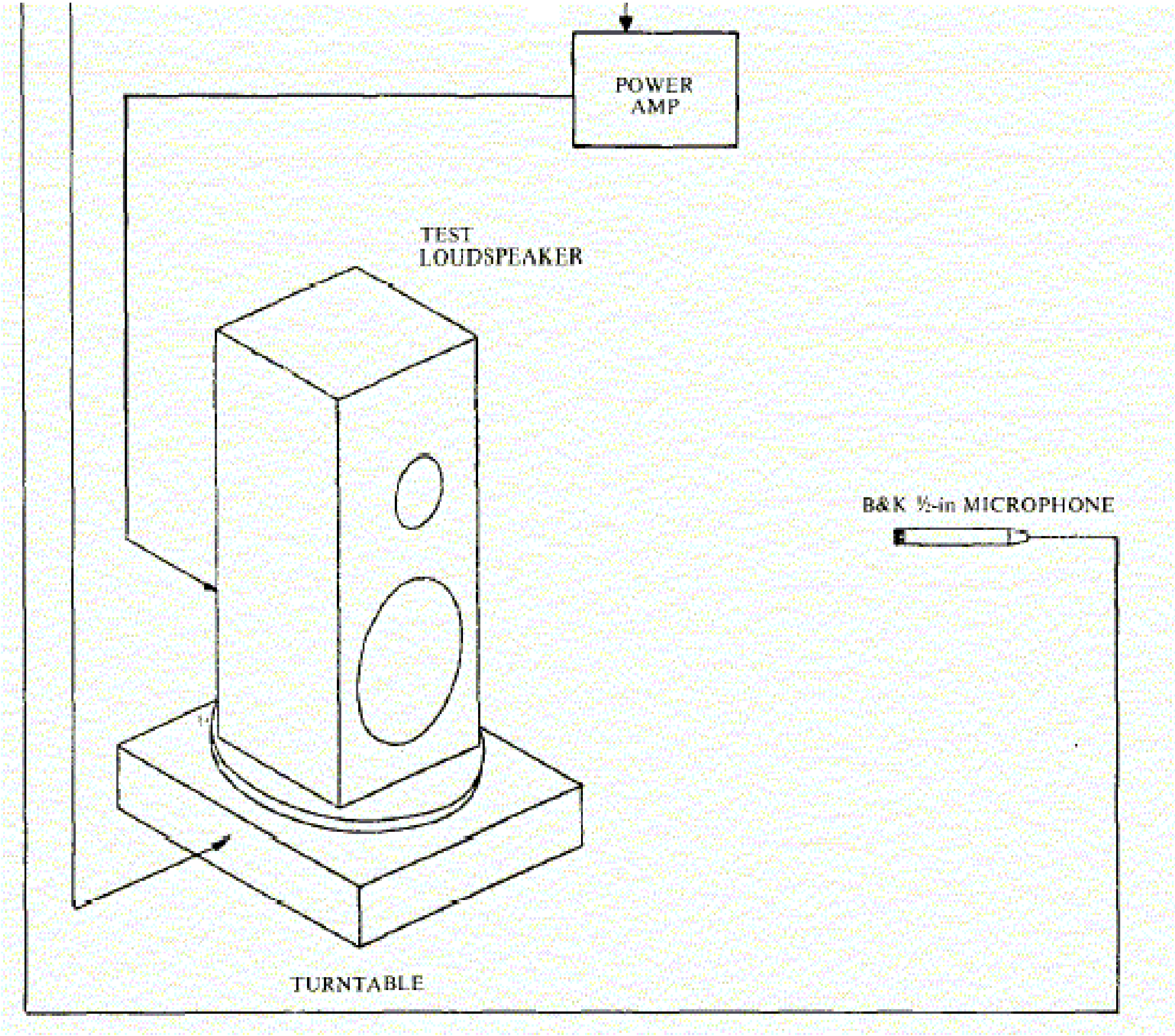
- Introduction
- Acoustic theory
- Some simulations
- Multipoles and the Acoustic Centre
- More simulations
- Measurements
- Discussion
- Conclusions

AES UK April 11,2006

Bowers & Wilkins

University of Waterloo

A typical setup for polar plots:
rotation axis through cabinet centre



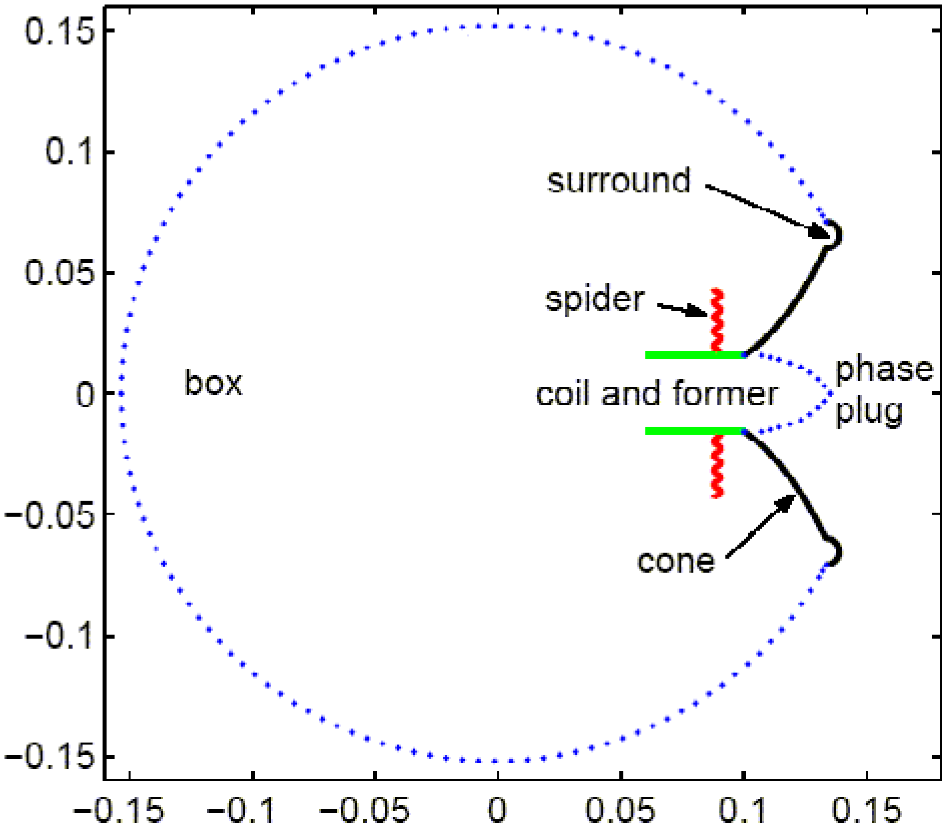
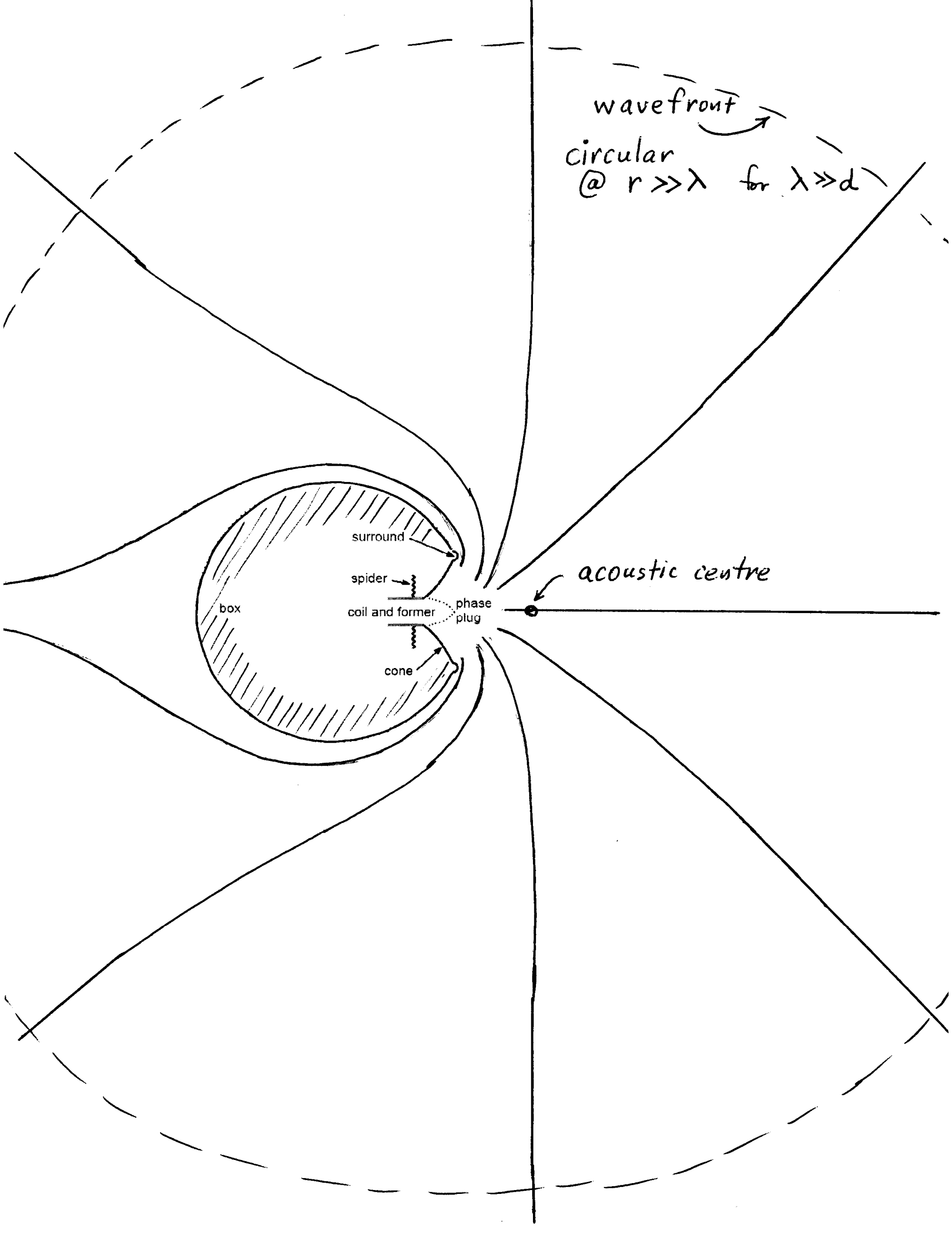


Fig. 1: The loudspeaker unit and box(enclosure)

(Recapitulating the original model)



Physical Intuition

The actual origin of the acoustic “flow” at low frequencies is well in front of the driver.

It is a good pivot point for polar plots.

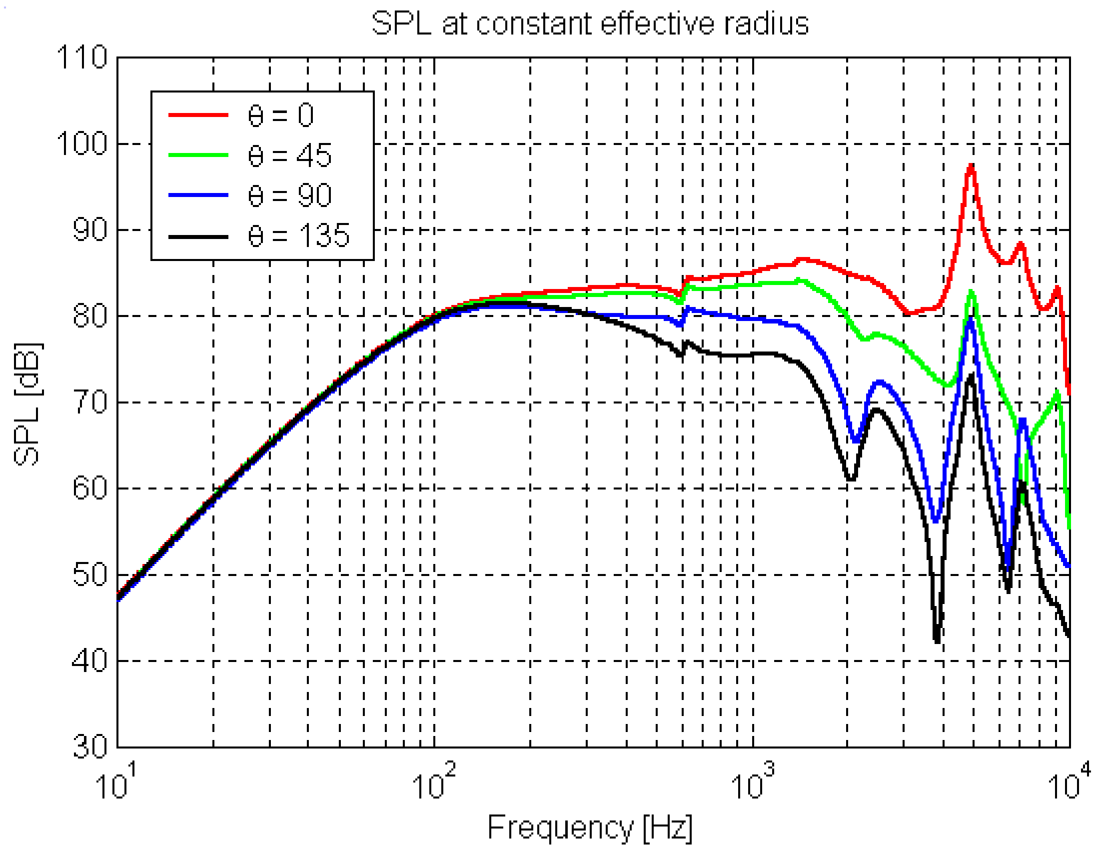


Figure 30. Output of the model loudspeaker at 1.5 m for different angles from the axis. The pivot point for the curves has been chosen to make the very low-frequency responses converge.

Last year's conclusions

The Acoustic Centre of a loudspeaker

- Boundary-elements calculate acoustic propagation
- At low frequency, 'flow' becomes simpler,
- and becomes spherical away from loudspeaker

- Plan: calculate acoustic output near loudspeaker
- Choose centre to make polar response uniform
- This empirical approach needs justification.

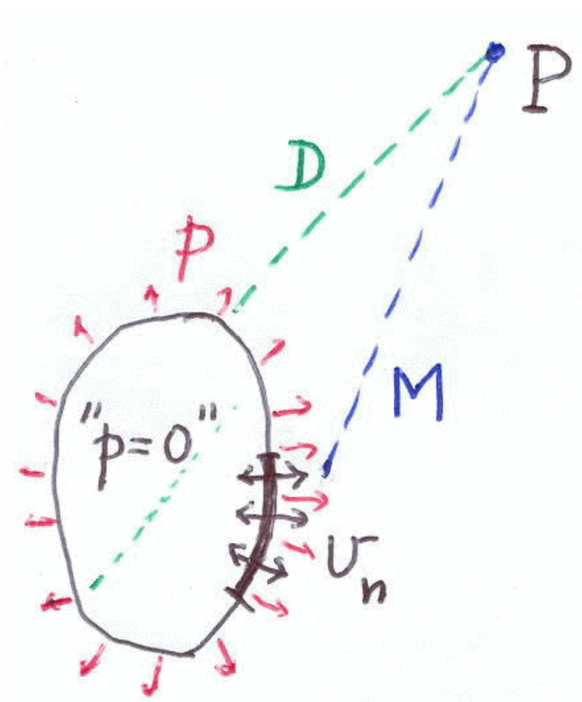
Acoustic Theory

The acoustic pressure $p(\mathbf{r},t)$ outside a source such as a loudspeaker can be expressed in terms of the normal component of the surface acceleration $\mathring{u}_n$ (the time derivative of the normal velocity u_n) and the pressure $p(\mathbf{r}_S,t)$ on the surface of the source, by the Kirchhoff-Helmholtz diffraction relationship:

$$p(\mathbf{r},t) = \iint [\rho\, \mathring{u}_n(\mathbf{r}_S, t-R/c) \,/\, (4\pi R)]\, dS$$
$$+ \iint [\mathbf{e}_R \cdot \boldsymbol{n}_S\, (\partial/\partial t + c/R)\, p(\mathbf{r}_S, t-R/c) \,/\, (4\pi cR)]\, dS,$$

in which the distance to the observation point is $R = |\boldsymbol{r} - \boldsymbol{r}_S|$, $\boldsymbol{e}_R$ is the unit vector in that direction, $\boldsymbol{r}_S$ is a vector on the surface, $\boldsymbol{r}$ is the vector observation point outside the surface, and $\boldsymbol{n}_S$ is the unit vector normal to the surface. For points inside the closed surface, the net result for $p(\mathbf{r},t)$ should be zero, and this condition allows computation of surface pressure from the source surface velocity.

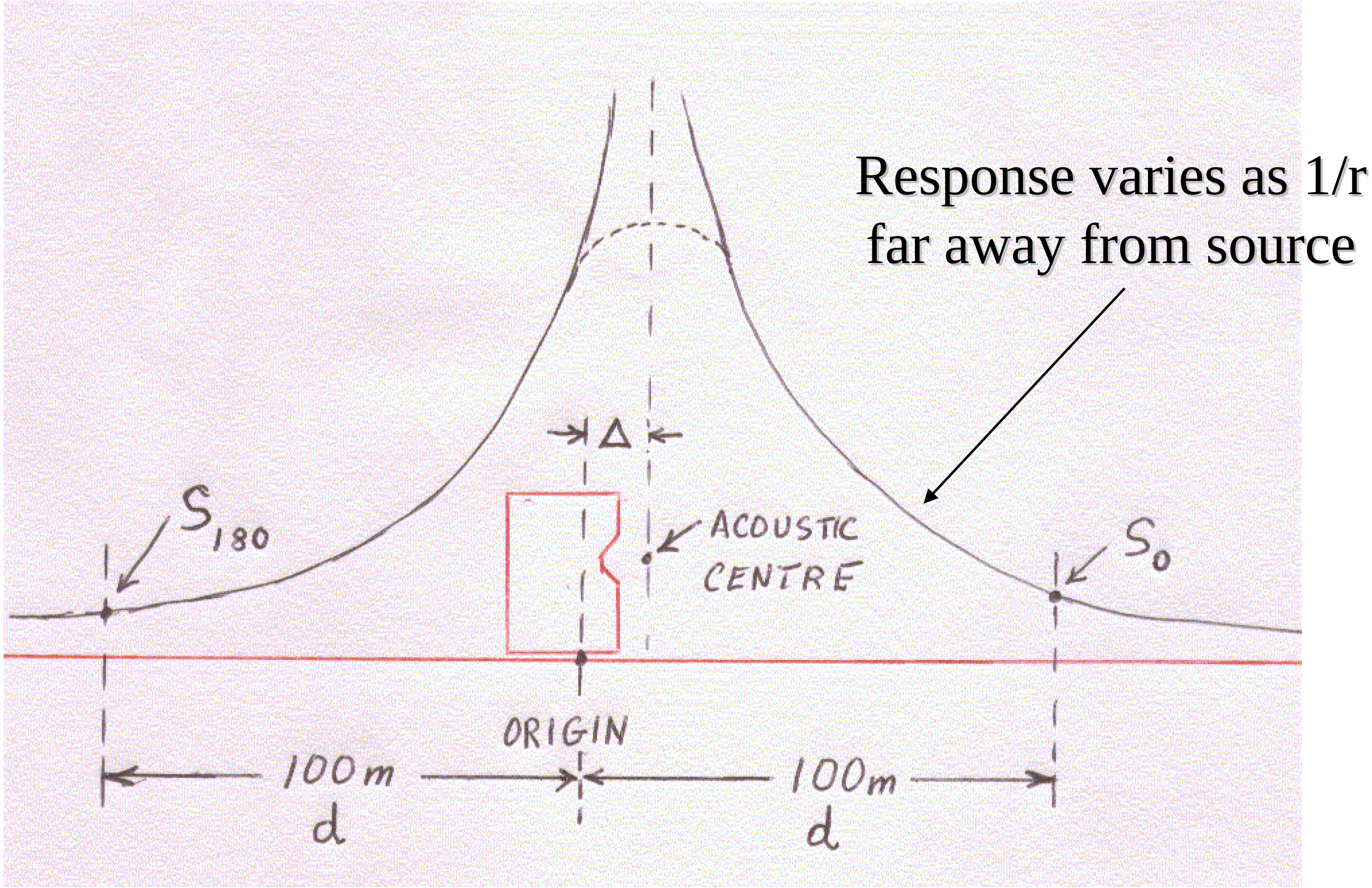
Boundary-Element Computations



The Kirchhoff-Helmholtz relationship for a discretised surface allows the pressure on all the boundary elements to be computed from the surface normal velocity by imposing the proper conditions on the surface.

Once computed, the monopole and dipole terms send out their signals as sources spread out over the surface of the loudspeaker, allowing calculation of the acoustic signal everywhere outside the loudspeaker.

Defining the acoustic centre at LF



How we calculate the acoustic centre

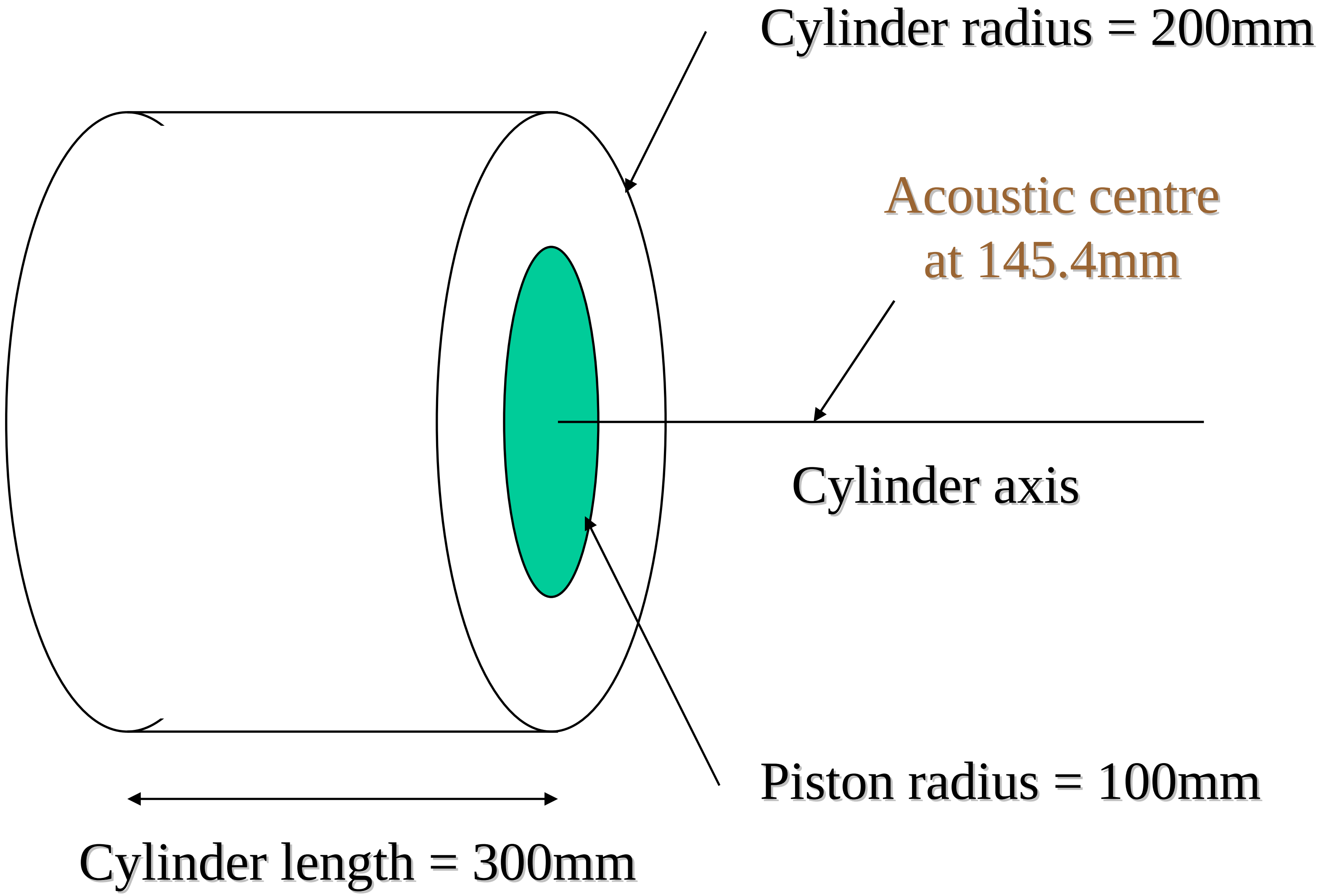
The basic assumption is that response must vary precisely as 1/r if correct point found.

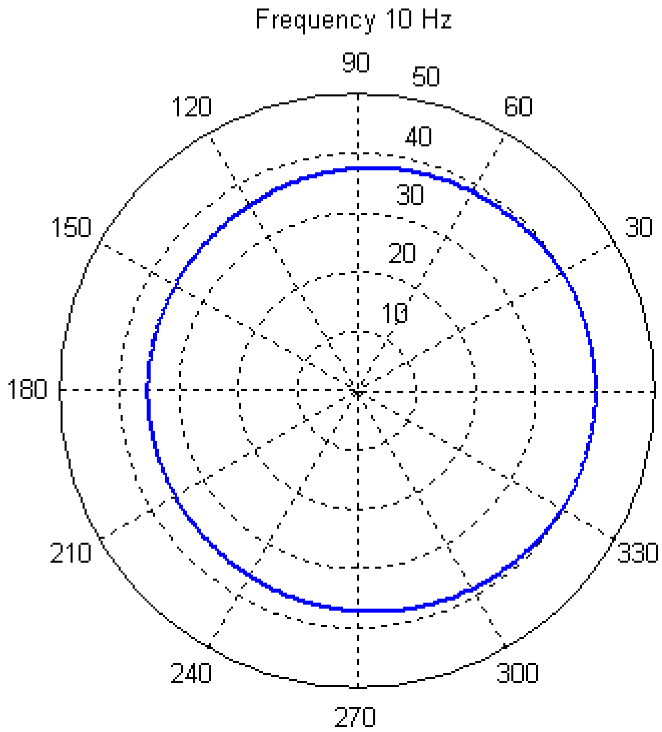
The amplitude response of the system was calculated at a large distance d=100m (the exact value doesn't matter) from the origin for 0° (on axis) and 180° (rear response), at a frequency such as 1 Hz, for which the wavelength is much larger than the cabinet size. At such distance and frequency, the amplitude response varies almost precisely as $1/r$, as stated above. Calling these responses S_0 and S_{180} respectively, the distance Δ from the origin to the true acoustic centre can then be shown to be

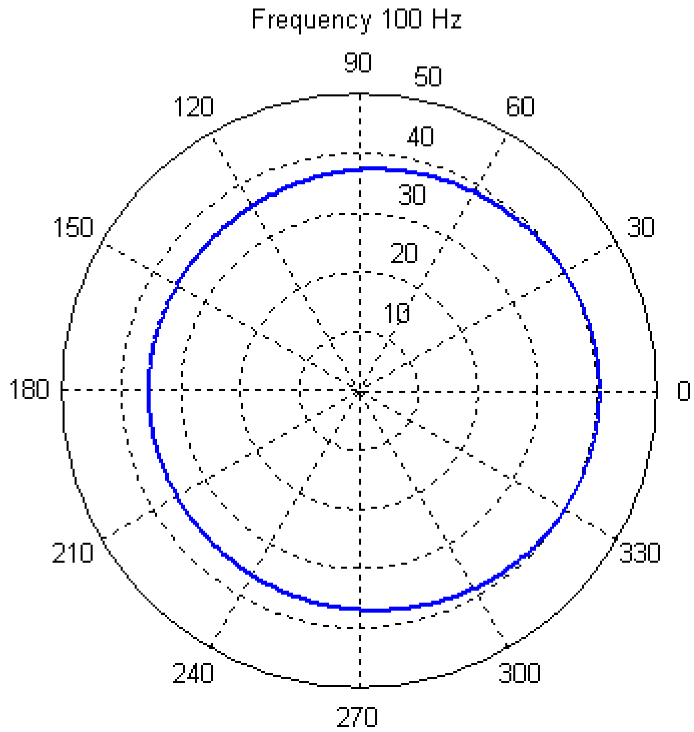
$$\Delta = d\,(S_0/S_{180}-1) \,/\, (S_0/S_{180}+1).$$

For our example cabinet, we find for d=100m that S_0 = –29.309890 dB and S_{180} = –29.345082 dB, which gives a value for Δ of 0.2026 m. It makes sense to refer the position of the acoustic centre to that of the baffle plane, so Δ must be corrected by subtracting 0.1332m from it, making the distance from the baffle to the acoustic centre δ = 0.0694 m, so δ/R=0.694 for R=0.1m

The axi-symmetric cabinet used in some of the simulations





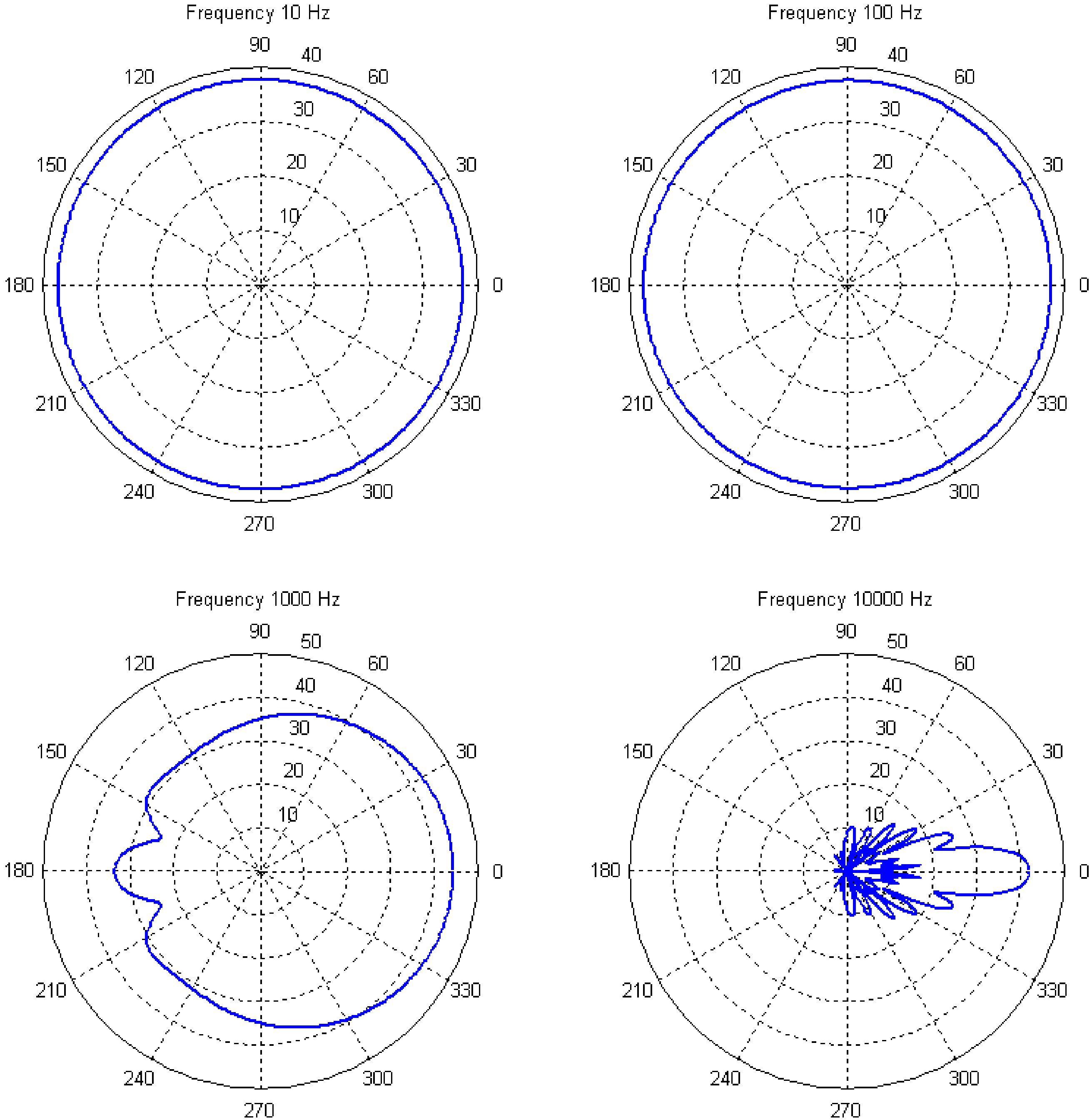


Cabinet
R=200mm
D=300mm
r=100mm

pivot point
–150mm
(middle of cabinet)

Obs_radius: 1.0m

Not at the Acoustic Centre

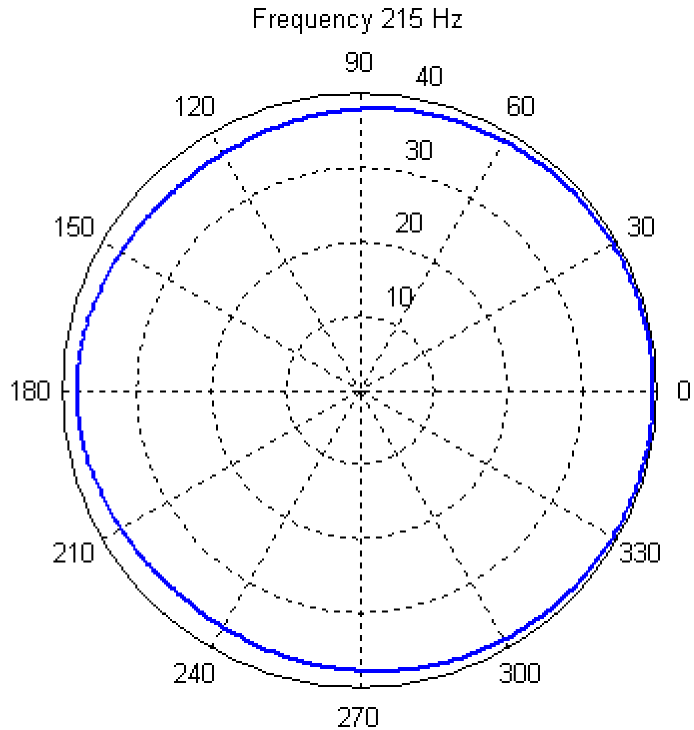


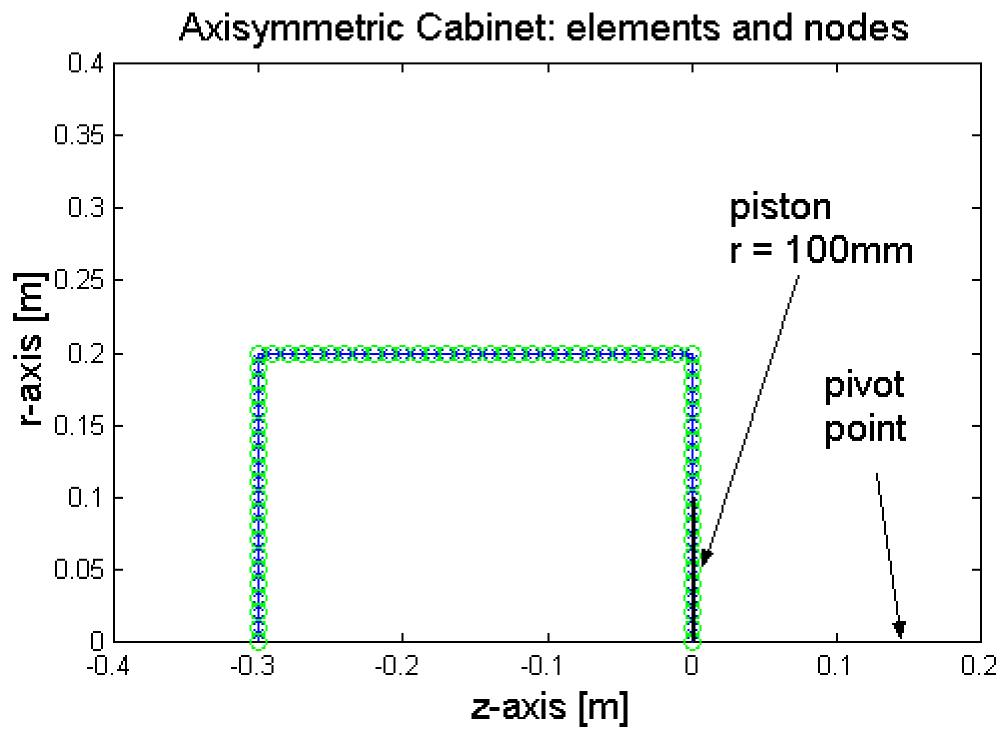
Cabinet
R=200mm
D=300mm
r=100mm

pivot point
145.4mm

Obs_radius: 1.0m

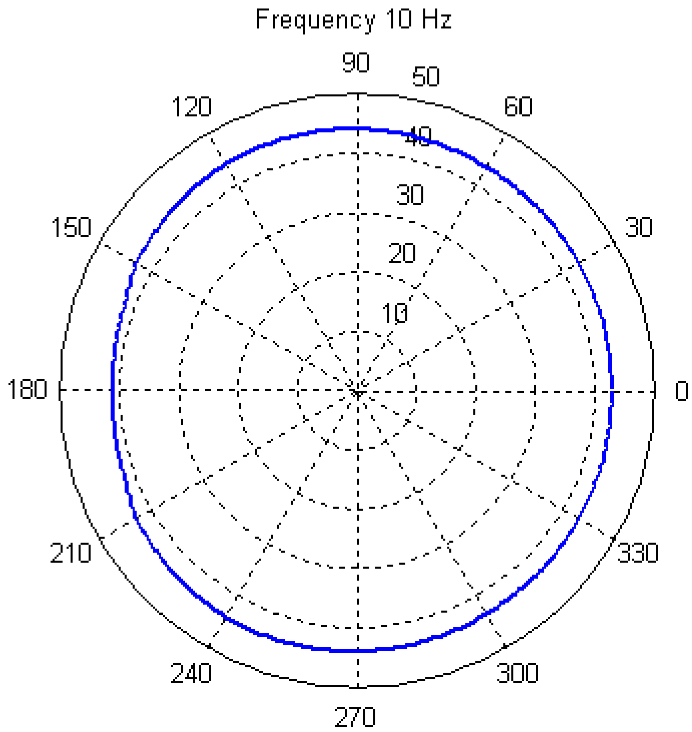
At the Acoustic
Centre

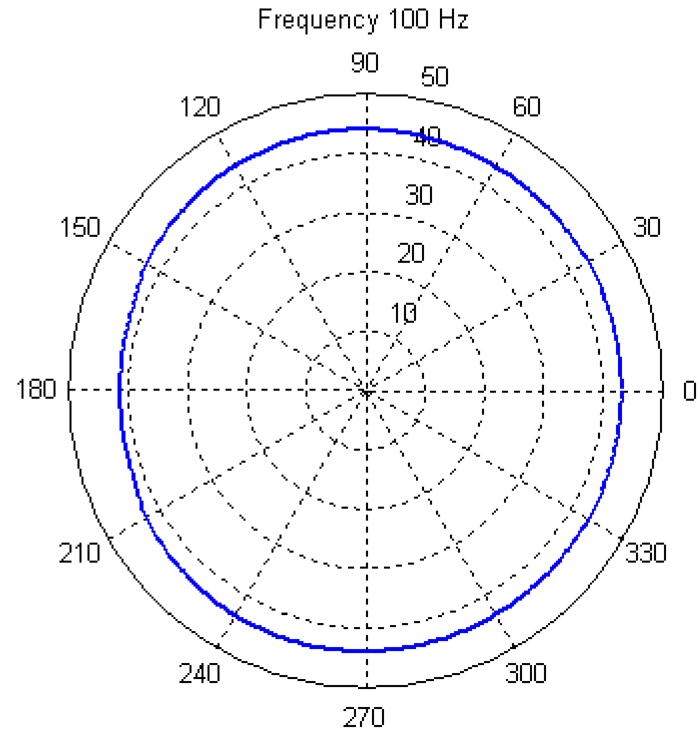




Even at 215 Hz the output is uniform within 2dB

At the Acoustic Centre





Cabinet
R=200mm
D=300mm
r=100mm

pivot point
145.4mm

Obs_radius: 0.5m

Even very nearby the response is omnidirectional

Expansion into Multipoles

If we consider now the loudspeaker to be a compact source in the vicinity of the origin, much smaller than a wavelength, and observe the system some distance away, the acoustic pressure can be manipulated into a multipole expansion [2] given by

$$p(\mathbf{r},t) = S(t-r/c)/r - \nabla\cdot\boldsymbol{D}(t-r/c)/r$$

$$+\Sigma\,(\partial^2/\partial x_\mu\,\partial x_\nu)\,Q_{\mu\nu}(t-r/c)/r + \cdots \qquad (3)$$

where the summation in the last term is over each of the three directions of the coordinate components x_μ and x_ν.

Composition of the Multipoles

The terms are defined by

$$S(t) = (\rho/4\pi) \iint \mathring{u}_n(\mathbf{r}_S,t)\, dS$$

Monopole

$$\boldsymbol{D}(t) = (1/4\pi) \iint [\rho\, \mathbf{r}_S\, \mathring{u}_n(\mathbf{r}_S,t) + \mathbf{n}_S\, p(\mathbf{r}_S,t)\,]\, dS$$

Dipole

$$Q_{\mu\nu}(t) = (1/8\pi) \iint [\rho\, x_{S\mu}\, x_{S\nu}\, \mathring{u}_n(\mathbf{r}_S,t) + (x_{S\mu}\, n_\nu + x_{S\nu}\, n_\mu)\, p(\mathbf{r}_S,t)]\, dS$$

Quadrupole

It is also clear that with an appropriate choice of origin, the dipole term

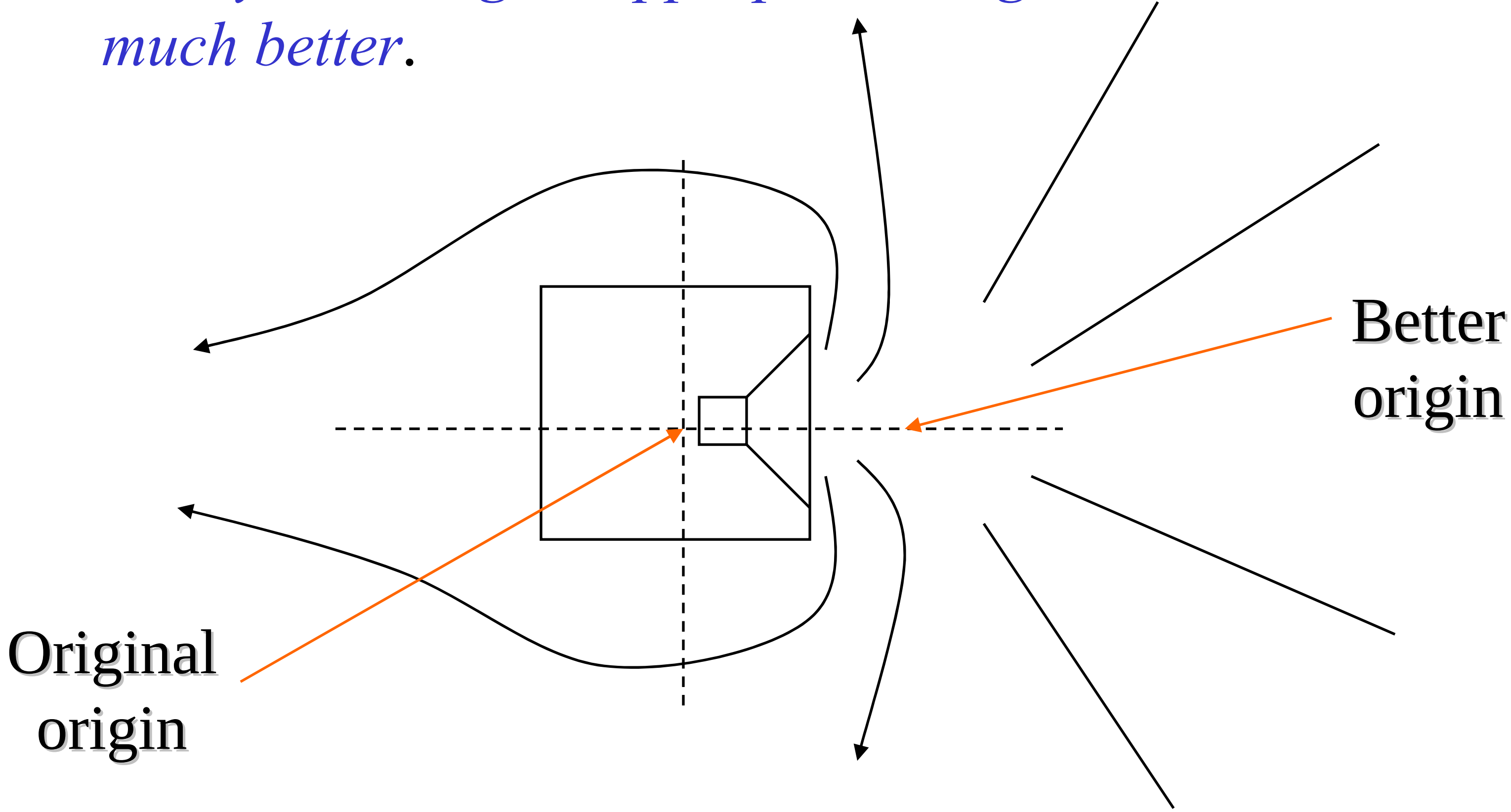
$$\boldsymbol{D}(t) = (1/4\pi) \iint [\rho\, \mathbf{r}_S\, \mathring{u}_n(\mathbf{r}_S,t) + \mathbf{n}_S\, p(\mathbf{r}_S,t)\,]\, dS$$

can be made to vanish.

And the higher terms are less
at higher frequencies and distances

It is clear that even without choosing a different origin, the *loudspeaker will always appear as a point source* if we get far enough away from it.

However, we may want to get relatively close to a loudspeaker when we measure its polar response, and *by choosing an appropriate origin, we can do much better*.



The acoustic centre concept

This choice of origin is our desired acoustic centre, since the system then looks at large distances like a point source, because the higher-order terms such as quadrupoles and higher-order multipoles are very much reduced at low frequencies and also have a faster fall-off rate as r increases. Consider a change in the origin describing the source to a position $\boldsymbol{r}_0$, such that the dipole term is then zero. We need to change only the vector $\boldsymbol{r}_S$ to $\boldsymbol{r}_S - \boldsymbol{r}_0$, since the functional dependences with $\boldsymbol{r}_S$ inside the integrals are dummy variables that simply mean we integrate over the source surface. Thus setting the dipole term to zero gives

$$0 = \iint [\rho\,(\mathbf{r}_S - \mathbf{r}_0)\,\mathring{u}_n(\mathbf{r}_S,t) + \mathbf{n}_S\,p(\mathbf{r}_S,t)\,]\,dS.$$

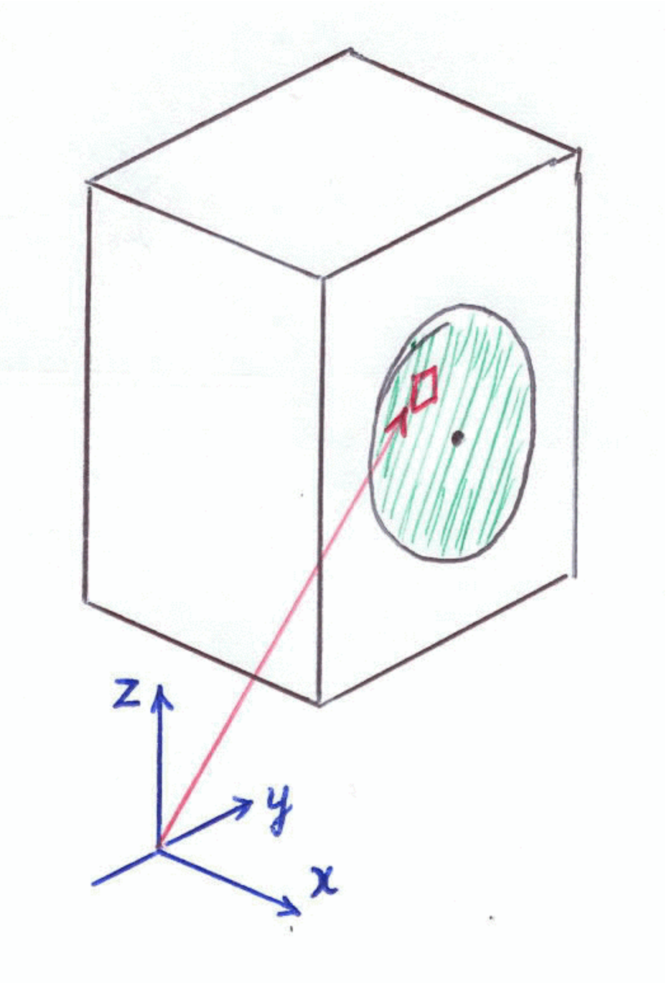
The integral of the normal acceleration over the surface with a constant vector $\boldsymbol{r}_0$ in the integrand is proportional to the total "volume acceleration" A

$$\iint \mathbf{r}_0\, \mathring{u}_n(\mathbf{r}_S,t)\, dS.= \mathbf{r}_0\, dQ/dt = \mathbf{r}_0\, A.$$

A is a scalar and has units m^3/s^2, while Q is the "volume velocity" (units m^3/s), so that we finally obtain for the acoustic centre $\boldsymbol{r}_0$

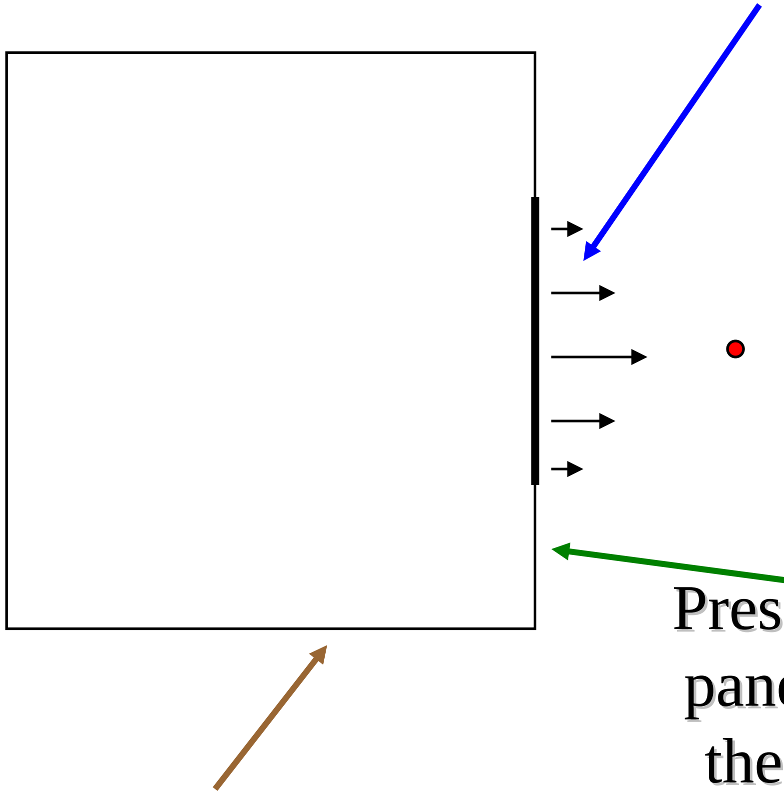
$$\mathbf{r}_0 = (1/A) \iint [\mathbf{r}_S\, \mathring{u}_n(\mathbf{r}_S,t) + \mathbf{n}_S\, p(\mathbf{r}_S,t)/\rho]\, dS.$$

The first term in this expression is the vector representing the position-weighted source strength. For our loudspeakers with circular cones this position will be the centre of the cone on its axis, which is not what we expect for the acoustic centre.



Dipole contributions to the acoustic centre

The surface pressure moves
the acoustic centre forward



Pressure on the stationary
panels of the box change
the acoustic centre also

Pressure on this panel causes
the acoustic centre to move sideways

We expect that the acoustic centre should be in front of the piston. It is the second term in the expression for $\boldsymbol{r}_0$ that makes this possible. The pressure $p(\mathbf{r}_S,t)$ on the piston and cabinet front baffle is in phase with the surface acceleration $\mathring{u}_n(\mathbf{r}_S,t)$ and has an associated forward-pointing unit vector $\boldsymbol{n}_S$ which moves the acoustic centre off the piston in the baffle plane and forward as expected. Physically, the pressure term (equivalent to dipole sources on the surface) represents forces which blow the air axially in phase with the simple source terms, and this causes the flow lines at large distances to appear to emanate from a point ahead of the piston.

$$p = \rho\,(dQ/dt)/(2\pi r) = \rho\,A/(2\pi r).$$

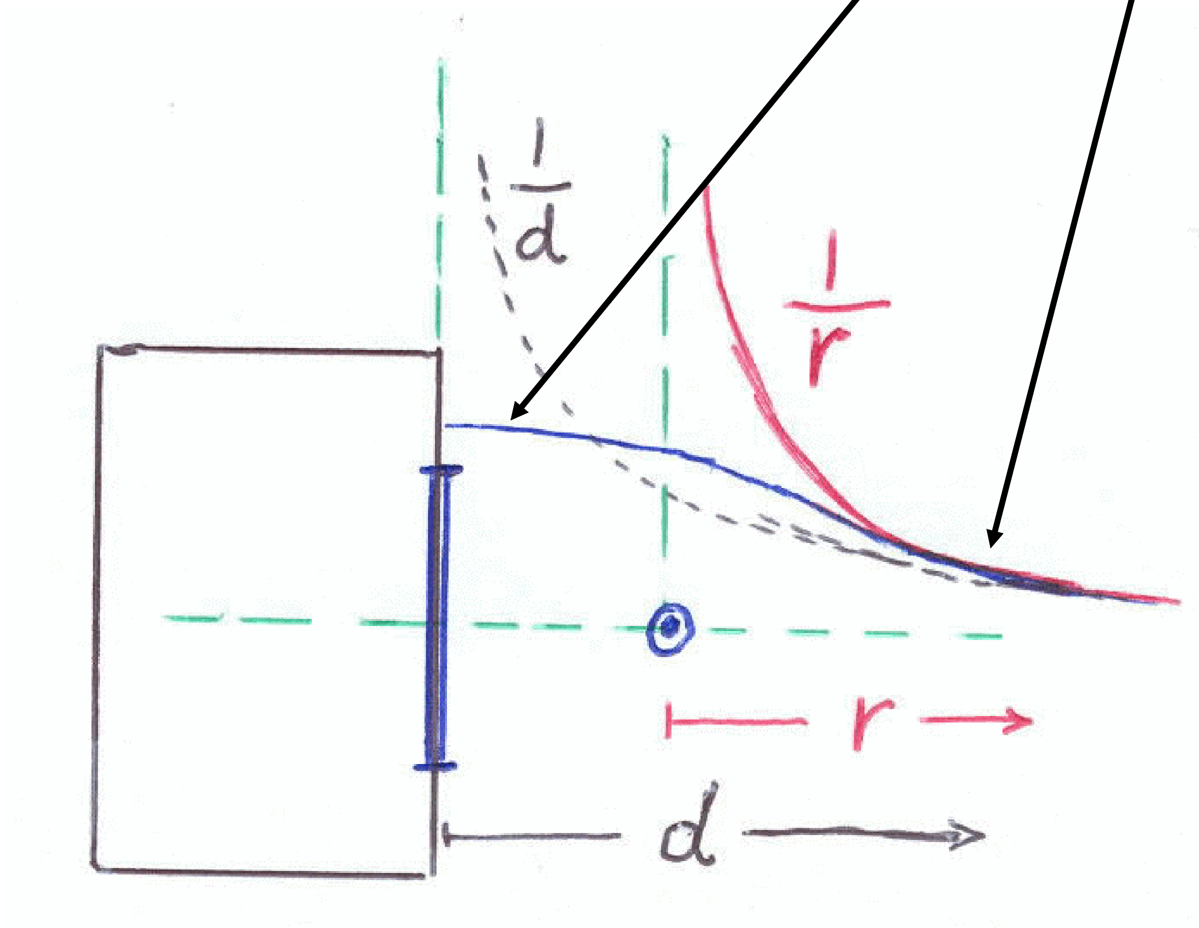
For a compact source the pressure is proportional to the volume acceleration, $A = dQ/dt$.

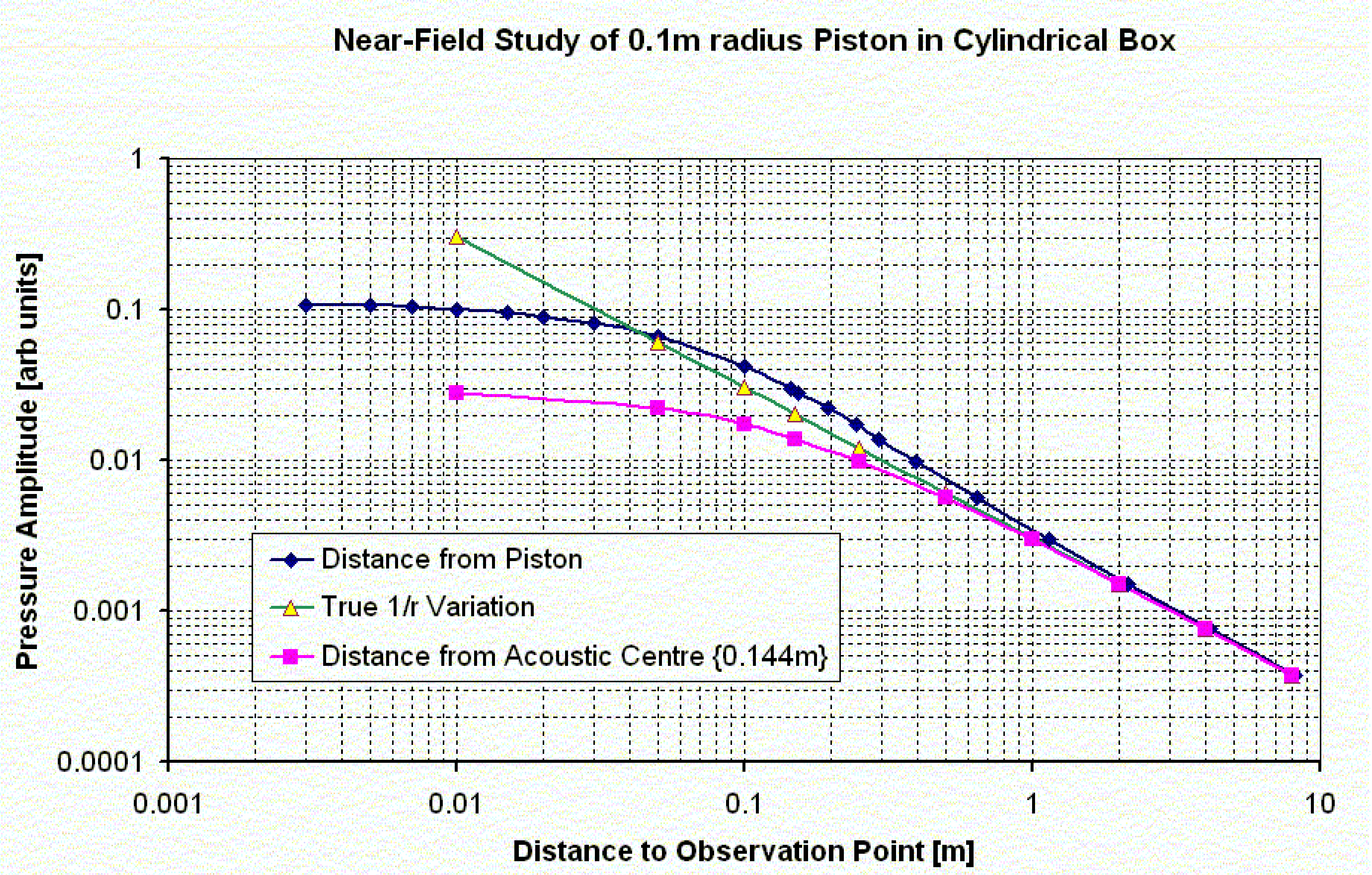
Using this the second integral in Eq.8 becomes

$$(1/A) \int [\mathbf{n}_S\, A/(2\pi r)]\; 2\pi r\, dr = \mathbf{n}_S\, a. \qquad (12)$$

The unit vector $\mathbf{n}_S$ points perpendicularly out from the disk, so the acoustic centre is predicted to be precisely one radius in front of the disk, on the disk axis. For now we will ignore the acoustic pressure on the rear surface of the disk. We can regard this as an upper bound to the real situation, since the front-surface pressure assumed in this calculation is actually too high. The baffle is quite finite and represents part of a cabinet. If we use the pressure from the point source as though it were in free space, representing too low an estimate, the denominator would be $4\pi r$ instead of $2\pi r$. This results in a lower bound for the acoustic centre being $a/2$ in front of the centre.

Actual pressure versus distance
to acoustic centre follows true 1/r
curve at moderate distance




Near-Field Study of 0.1m radius Piston in Cylindrical Box
Pressure Amplitude [arb units]
1
0.1
0.01
0.001
0.0001
0.001
0.01
0.1
1
10
Distance to Observation Point [m]
Distance from Piston
True 1/r Variation
Distance from Acoustic Centre {0.144m}

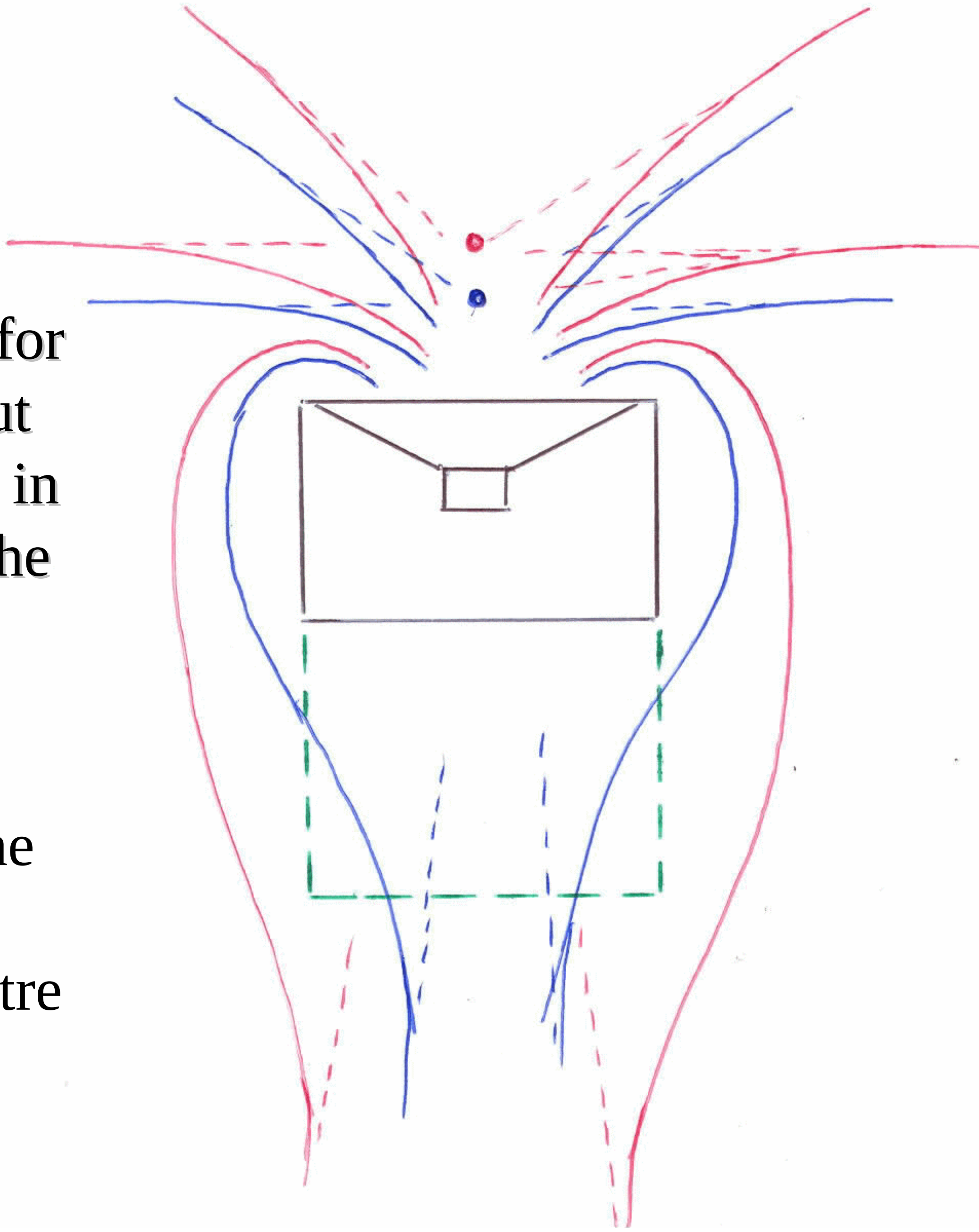
Theory has shown us:

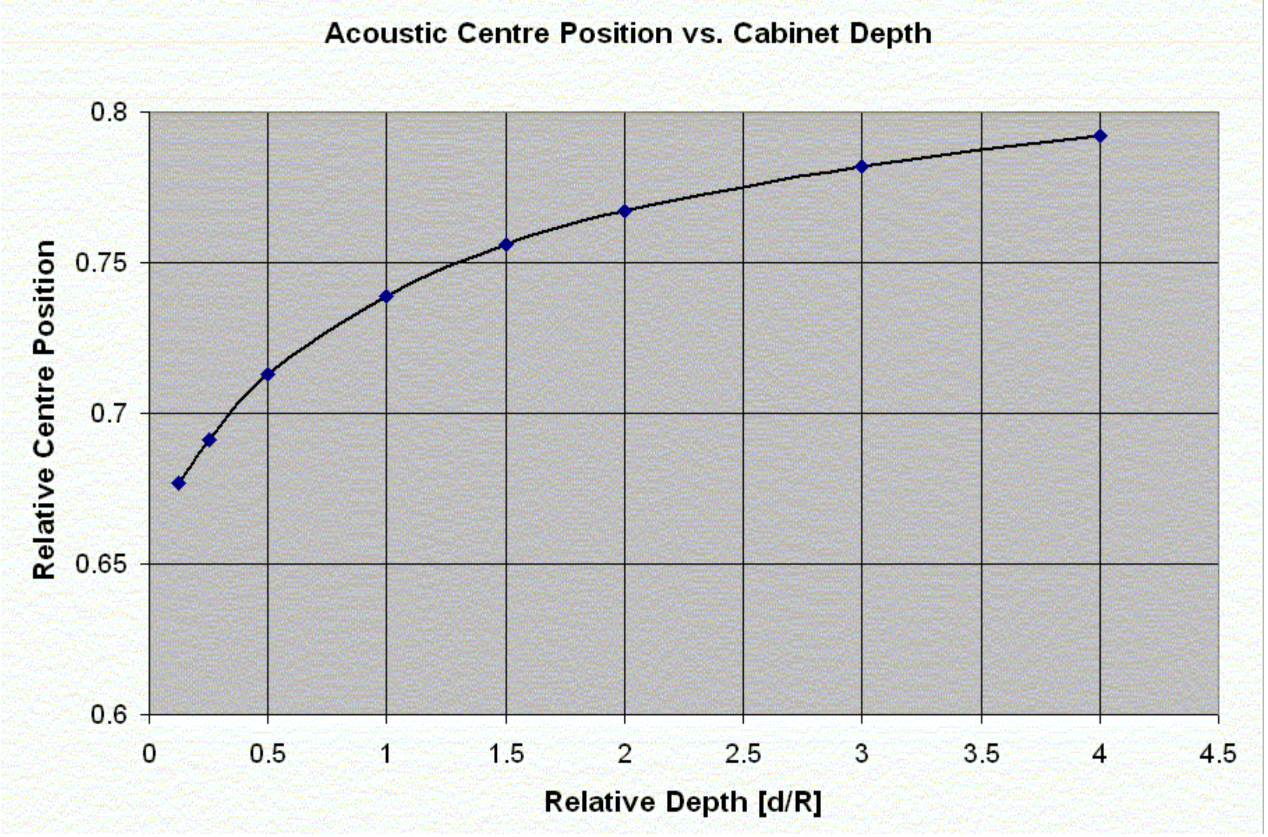
- dipole terms move forward the centre
- typical distances from piston are ~d_{baffle}
- pressures over the whole cabinet count
- works for LF and compact source

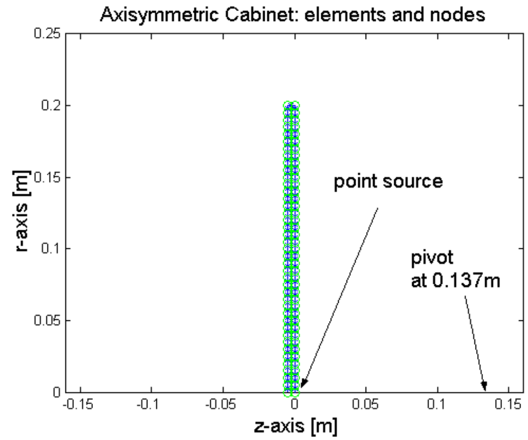
- we need to check these predictions
- with simulations and measurements

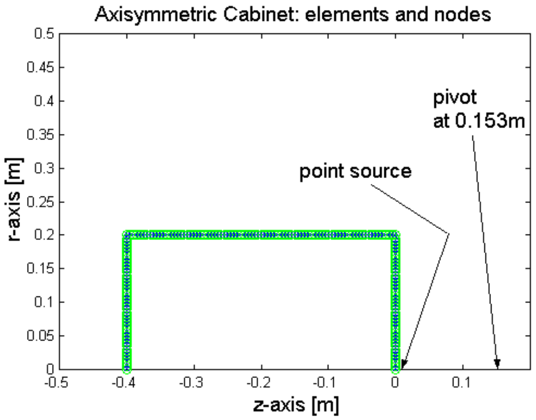
Effect of Cabinet Depth

Note how the flow lines in blue for the shorter cabinet are pushed out by a cabinet extension, resulting in an acoustic centre further from the cone in red. Increase in cabinet depth causes that space to be excluded from the air flow, and any exclusion of space behind the baffle pushes the air forward, thereby moving the acoustic centre forward also.




Acoustic Centre Position vs. Cabinet Depth
Relative Centre Position
Relative Depth [d/R]
0.8
0.75
0.7
0.65
0.6
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

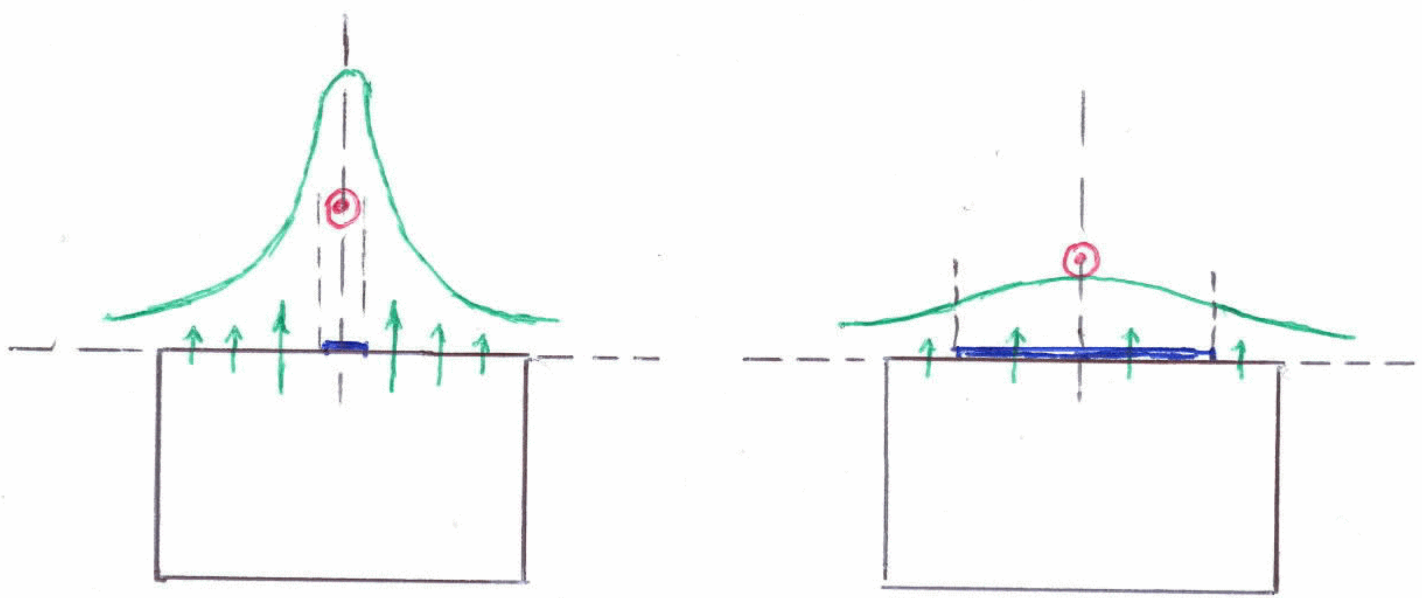

Axisymmetric Cabinet: elements and nodes
point source
pivot
at 0.137m
r-axis [m]
z-axis [m]

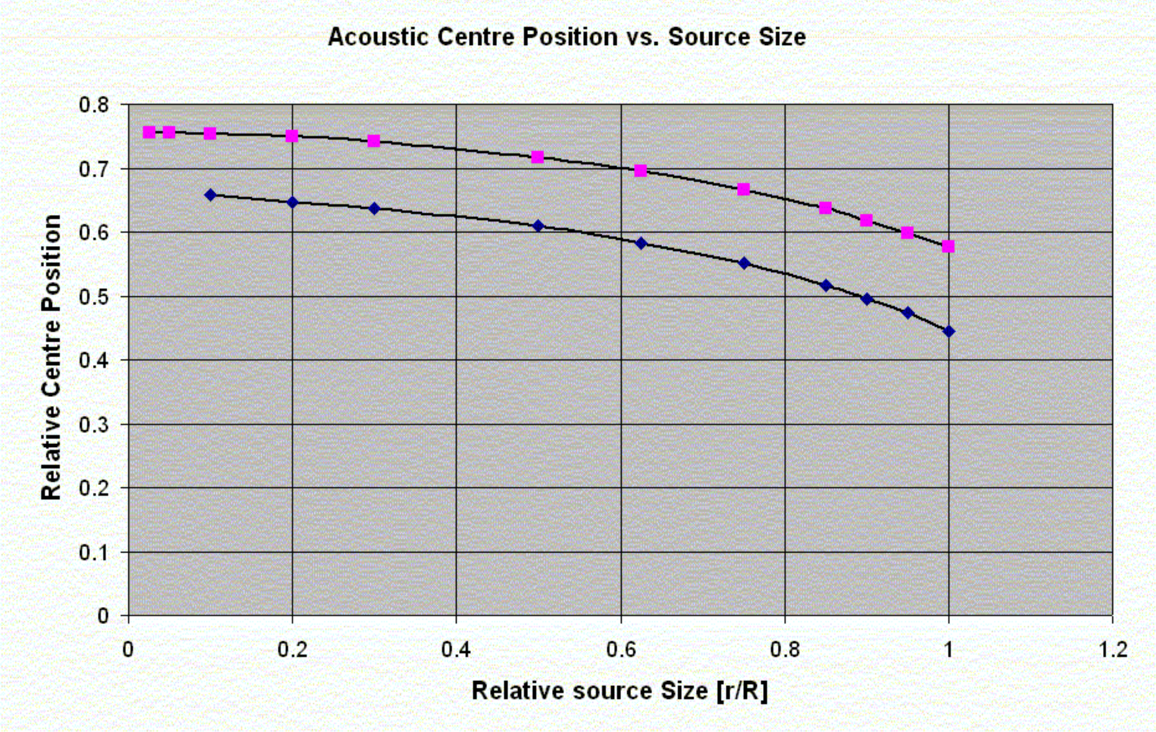

Axisymmetric Cabinet: elements and nodes
pivot
at 0.153m
point source
r-axis [m]
z-axis [m]

Effect of Source Size

When source size is increased and the output volume velocity is kept the same as for a smaller source:

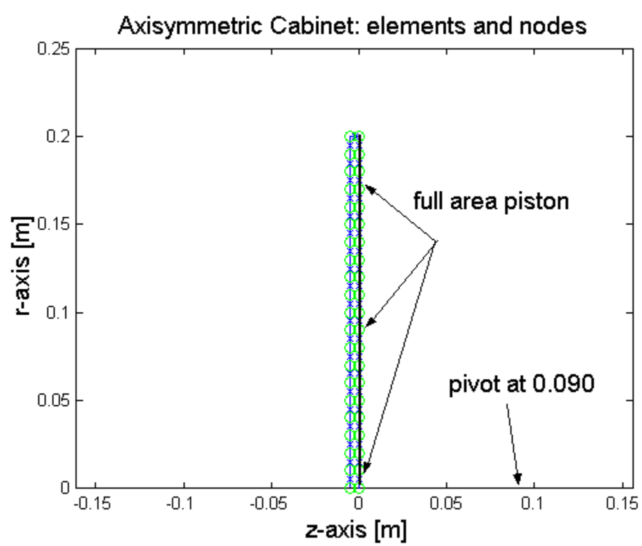
-- the volume acceleration term is not changed
-- the pressure term is decreased since the pressure is less concentrated, which means the larger source has less dipole term, thus a closer acoustic centre

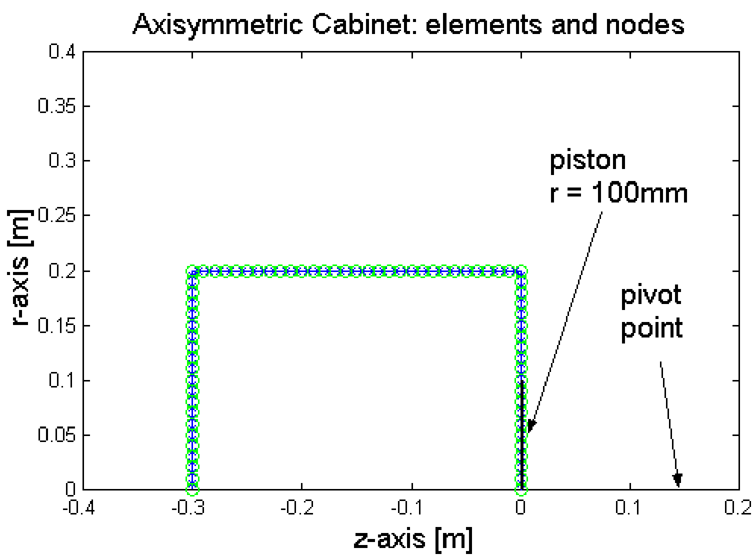



Acoustic Centre Position vs. Source Size
Relative Centre Position
Relative source Size [r/R]
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2

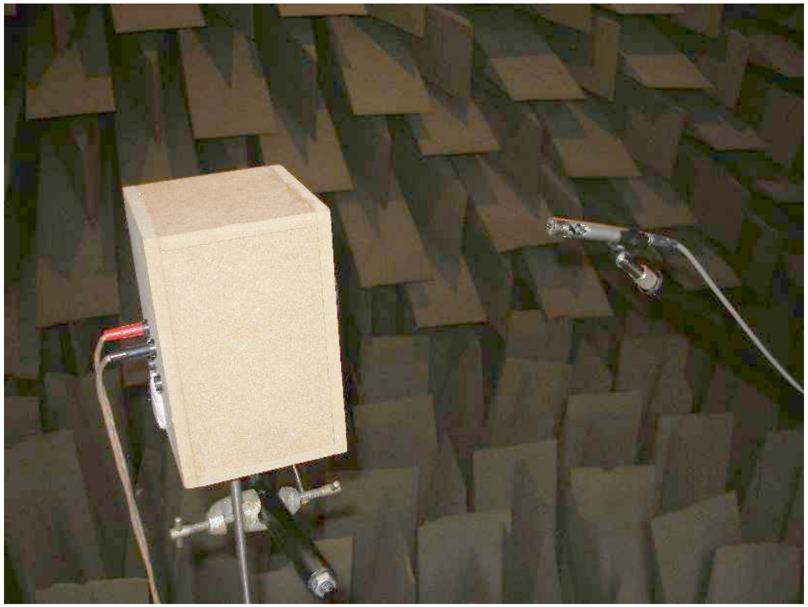
d/R=1.5

d/R~0


Axisymmetric Cabinet: elements and nodes
full area piston
pivot at 0.090
r-axis [m]
z-axis [m]


Axisymmetric Cabinet: elements and nodes
piston
r = 100mm
pivot
point
r-axis [m]
z-axis [m]

Some measurements to check the concept



On axis

180 degrees

These two orientations are pivoted about a vertical axis just behind the centre of the cabinet, a typical pivot point. The microphone is 22 cm from this axis. The box is 11cm deep.

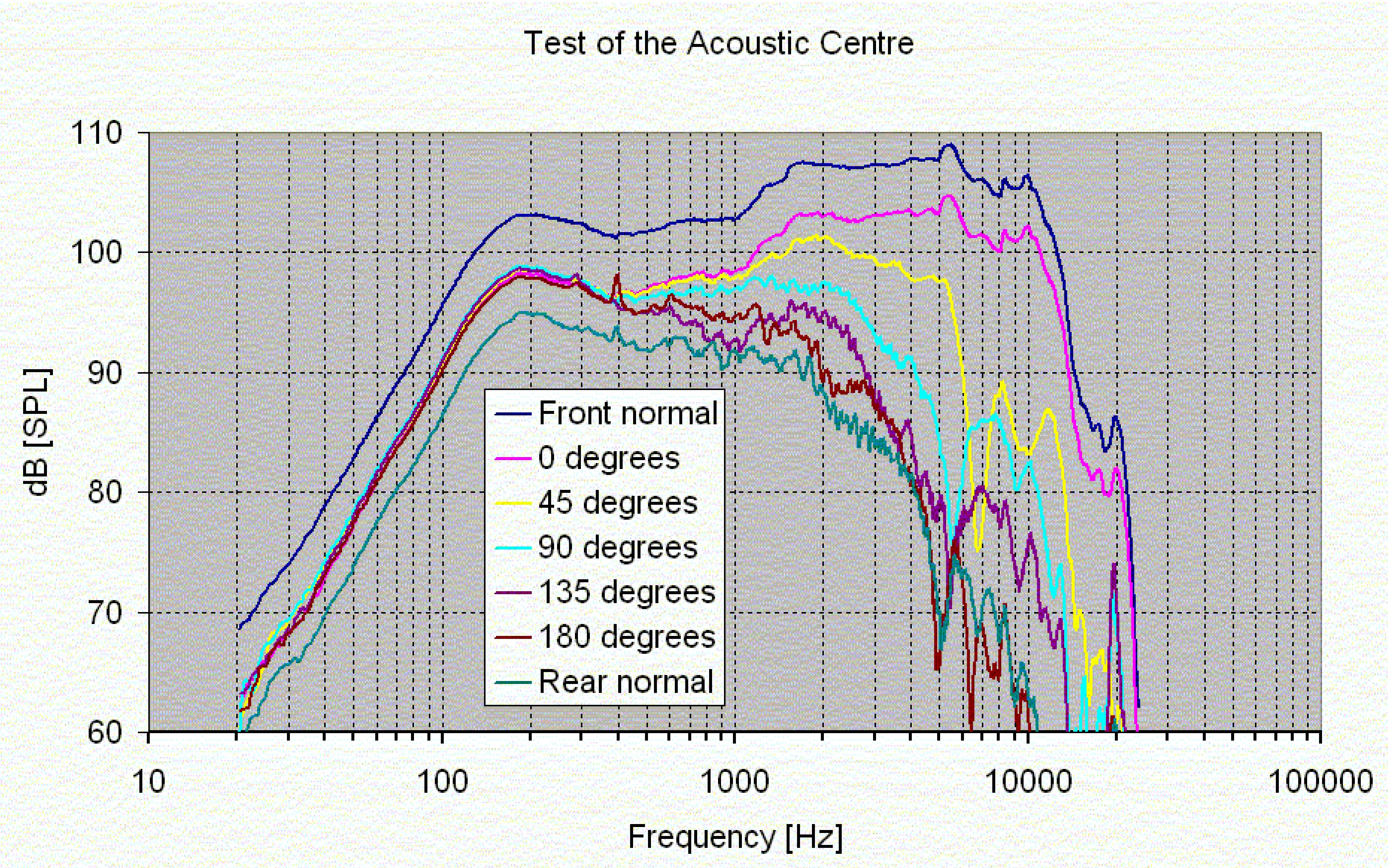
On axis

90 degrees

180 degrees

Showing a variety of loudspeaker orientations pivoted about the acoustic centre. The acoustic centre is located directly above the lower vertical support rod, and the microphone is 22 cm from the line defined by the rod.

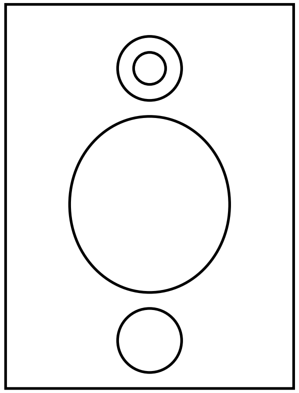
At 180 degrees the microphone is only about 6 cm from the back of the cabinet!



Test of the Acoustic Centre
110
100
90
80
70
60
dB [SPL]
Front normal
0 degrees
45 degrees
90 degrees
135 degrees
180 degrees
Rear normal
10
100
1000
10000
100000
Frequency [Hz]

What happens when there is a port?

It acts as a second source, but its frequency response is different than the main unit…

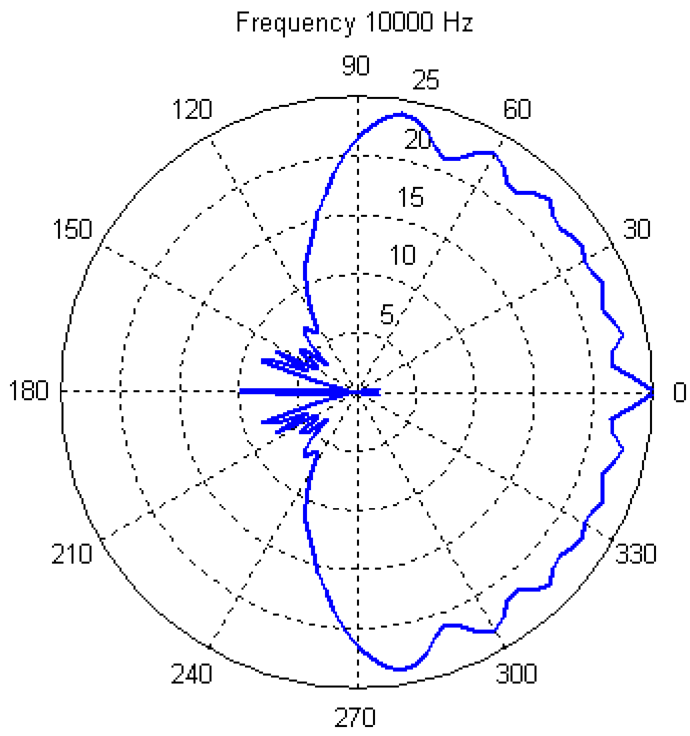


**If the port is on the front baffle,
the acoustic centre for it has a similar distance,
except for size, which has a small influence,
so the net result will not be that different.**

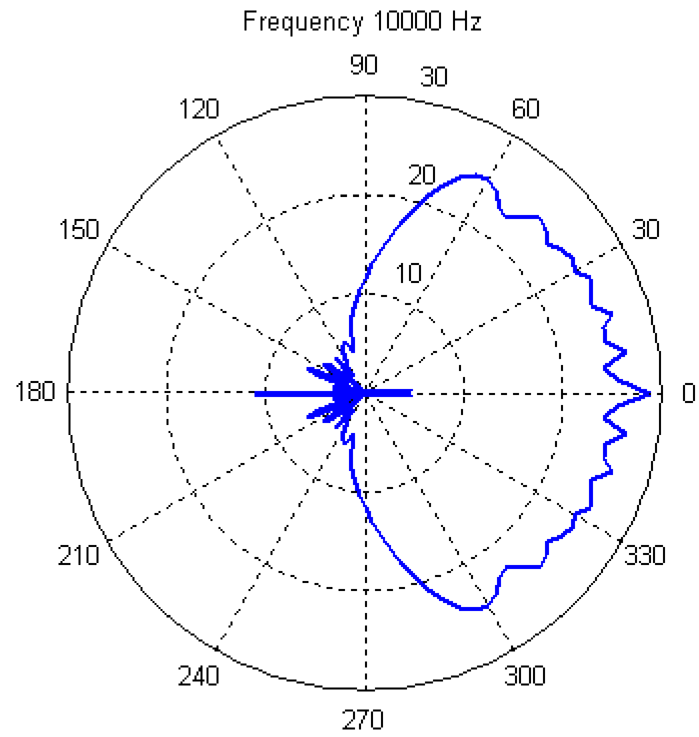
**If the port is on the rear,
for frequencies in the port range,
the acoustic centre will be that of two sources,
so it lies on a line joining them, at a balance point,
which follows directly from theory.**

How are the higher frequencies affected by the choice of pivot point?

R=200mm, D=400mm, pt source, Obs_radius=1.0m



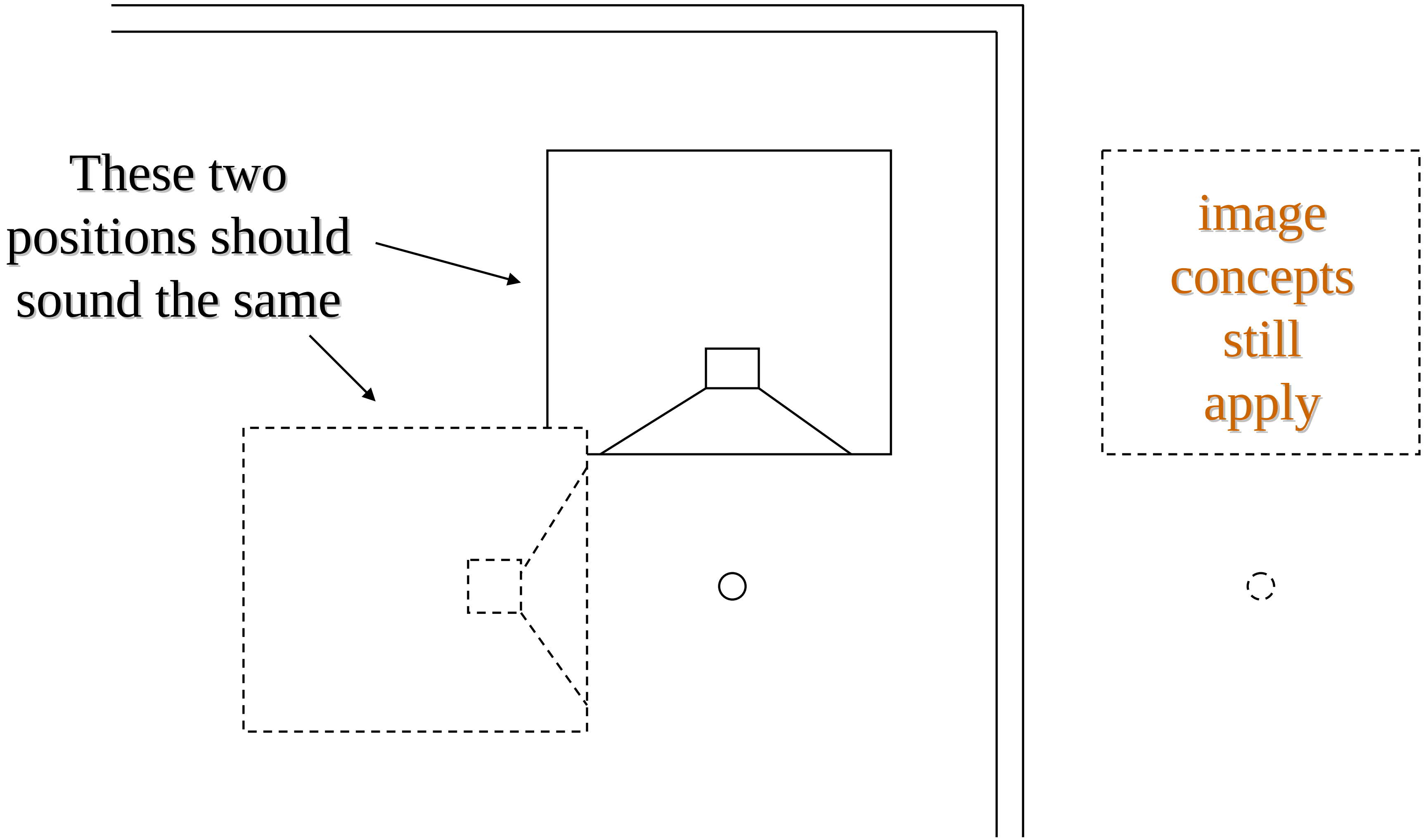
Pivot point 153.1mm



Pivot point -200mm

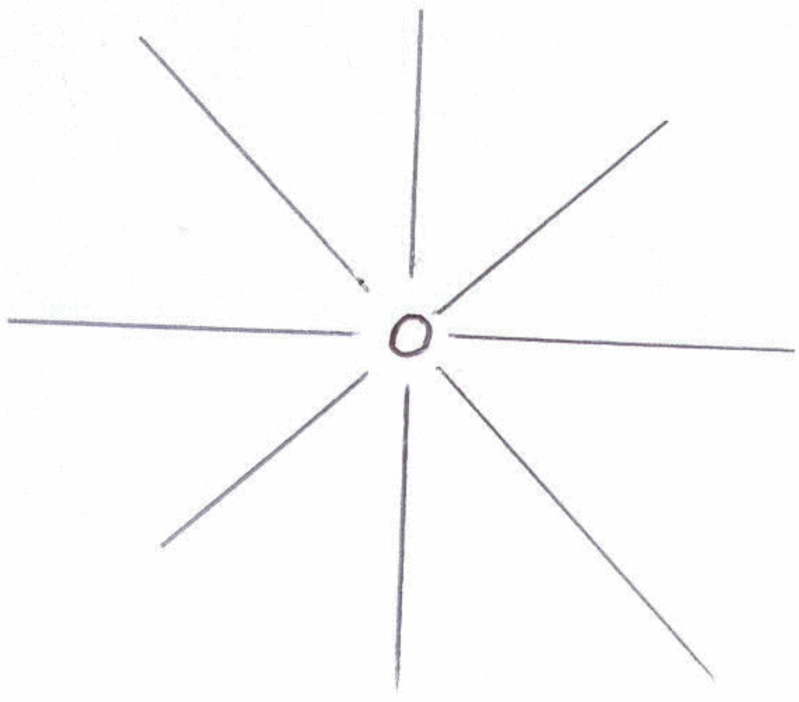
The basic difference is simple vignetting

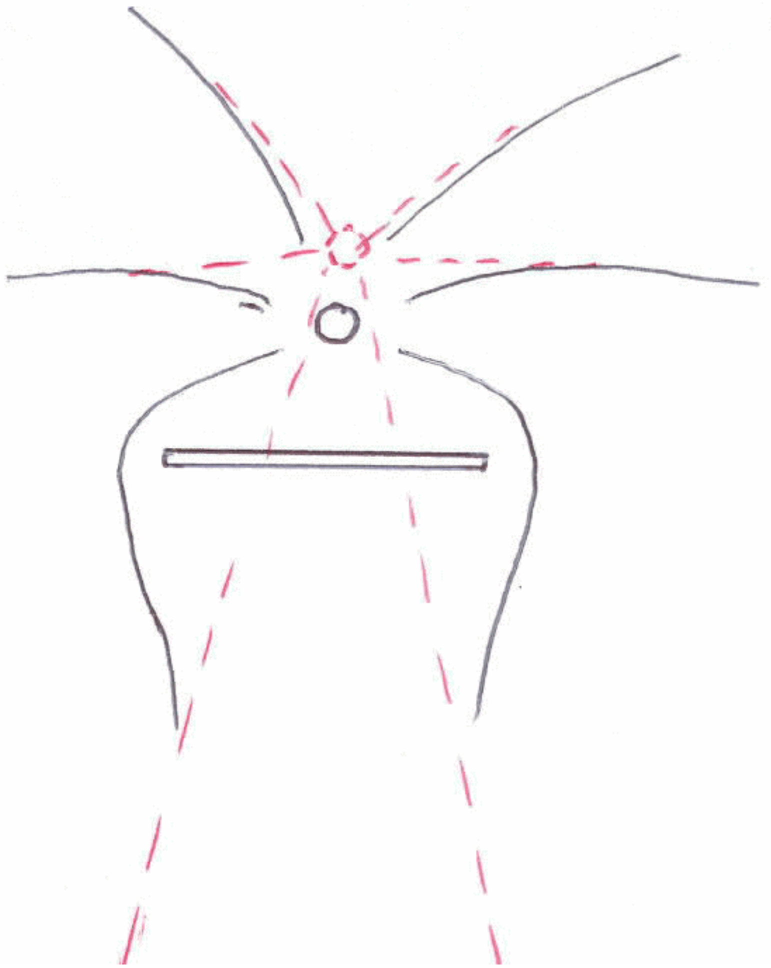
Subwoofers and their placement



Effect of an obstruction

Question: Where is the temporal acoustic centre?





Answer: It *really is* at the acoustic centre.

- In front, time is advanced,
  - Since the cabinet concentrates forward air
- In the rear, time will be retarded,
  - Since path is lengthened, air diverted

- At higher frequencies, this action ceases,
  - And the acoustic centre is close to the cone.
- Question: Is this variation minimum-phase?

General conclusions:

- From simulations, the acoustic centre concept works
- The distance from driver to acoustic centre:
  - decreases with increasing driver size
  - increases with increase of baffle size
  - increases with longer cabinets
- The concept is valid to midband frequencies for typical cabinets
- Theory supports the concept, and measurements
- Reflex ports complicate things
- A number of special points were made

Thanks for your attention!